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Magnetic fields and accretion discs around static black holes

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Abstract. We investigate some aspects of accretion onto static black holes immersed in a uniform magnetic field. The Ernst metric is employed to find the 'Keplerian' angular momentum distribution and the efficiency of mass-to-energy conversion for a plasma and for test particles. Under almost all physically reasonable conditions for hydrodynamic accretion the effect of the magnetic field is small. However, for test particles the effect can be very important and the efficiency can approach unity.

1. Introduction

Accretion discs around massive black holes are the leading candidates for the powerhouses of active galactic nuclei (for a review see Pringle 1981). A recently developed class of thick accretion disc models can provide for luminosities substantially above the Eddington limit and can also yield an attractive way to collimate and accelerate beams of radiation and matter (Lynden-Bell 1978, Paczyński and Wiita 1980 (hereinafter referred to as PW), Jaroszyński *et al* 1980, Abramowicz and Piran 1980, Sikora and Wilson 1981). This may be of particular importance in view of the frequency with which small-scale jets are found in quasars and the nuclei of radiogalaxies (e.g. Readhead and Wilkinson 1980).

These thick discs have their inner edges somewhere between the last stable circular orbit for particles, $r_{ms}(=3r_g=6m)$, and the marginally bound orbit, r_{mb} $(=2r_g=4m$, for a Schwarzschild black hole of mass m, and we use units where $G=c=1$). They must exhibit a non-Keplerian angular momentum distribution in their inner regions (e.g. PW). Although magnetic fields will certainly be present around the black hole and in the accreted material (Bisnovatyi-Kogan 1979), very little attention has been paid to the question of how magnetic fields may affect the values of r_{mb} and r_{ms}. We expect that if r_{mb} can be decreased, then the size of the thick discs can be increased (PW) and the opening angle of the 'funnels' or 'vortices' can be reduced (Abramowicz and Piran 1980), thus increasing both luminosity and collimation. If r_{ms} can be reduced, then the efficiency of the conversion of rest-mass to energy may be expected to increase, thus partially counteracting the unfortunate tendency of fatter, more supercritical, discs to be less efficient in converting mass into radiation (PW, Jaroszyński *et al* 1980).

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In this paper we shall present a calculation based upon the idealisation of the Ernst (1976) static space–time which is supposed to describe a Schwarzschild black hole immersed in a uniform magnetic field, with $|Bm| \ll 1$, so that the mass–energy of the magnetic field is small compared with the mass of the black hole. Dadhich *et al* (1979, hereinafter referred to as DHV) have analysed the trajectories of charged particles in the Ernst metric and have shown that bound orbits always exist for realistic magnetic field strengths. We shall use the DHV effective potential to find the 'Keplerian' angular momentum per unit mass as a function of equatorial radius. We will then find analytic approximations for r_{ms} and r_{mb} valid in the hydrodynamic plasma accretion and in the test particle limits, and will also present some numerical results comparing the efficiency of accretion at r_{ms} with that for the pure Schwarzschild metric.

The work most closely related to ours is that of Prasanna and collaborators (Prasanna and Varma 1977, Prasanna and Chakraborty 1981; for a review see Prasanna 1980) who have analysed the motion of charged particles in Schwarzschild and Kerr metrics with dipole magnetic fields added externally. Their results are qualitatively similar to ours, but they do not explicitly look for the Keplerian distribution of r_{ms}, nor do they examine astrophysical constraints on allowed values of B or the charge of the accreted material, as we do in § 3 below. Although our approach is much cruder than the detailed self-consistent solutions of the Einstein–Maxwell equations used to construct models of charged rotating discs (Prasanna and Chakraborty 1981), we trust that the new, albeit tentative, conclusions we draw are of some interest. Other authors have considered electromagnetic effects near discs to generate charged particle beams (Lovelace 1976) and also to extract energy from a Kerr black hole (Blandford and Znajek 1977, Znajek 1977). These interesting calculations are only tangentially related to ours, but certainly deserve further development and comparison with the recent observations of jets in galactic nuclei.

2. Effective potential and 'Keplerian' angular momentum

Ernst's (1976) metric is given by

$$ds^2 = \lambda^2[(1-2mr^{-1})^{-1}\,dr^2 + r^2\,d\theta^2 - (1-2mr^{-1})\,dt^2] + [r^2(\sin^2\theta)/\lambda^2]\,d\phi^2 \tag{2.1}$$

$$\lambda = 1 + B^2r^2\sin^2\theta \tag{2.2}$$

$$\Phi = Br^2(\sin^2\theta)/\lambda \tag{2.3}$$

where Φ is the electric potential with respect to the Killing vector $\eta^i\,(\partial\phi)$ and B is the value (assumed constant) of the magnetic field along the axis.

The effective potential can be easily found for motions confined to the equatorial plane, so with $\theta = \frac{1}{2}\pi$ and the momentum in the θ direction equal to zero we have

$$V = \lambda^2(1-2mr^{-1})[\mu^2 + (\lambda^2/r^2)(l-e\Phi)^2] \tag{2.4}$$

(DHV, equation (13)) for a particle of mass μ and charge e, where the angular momentum is given by

$$l = \lambda^{-2}\mu r^2(\sin^2\theta)\dot{\phi} + e\Phi \equiv \psi + e\Phi \tag{2.5}$$

(the λ^{-2} factor was inadvertently dropped in DHV, equation (10)).

We calculate the angular momentum distribution characterised by a balance between rotational, gravitational and electromagnetic forces (but neglect radiation losses) in analogy with the Keplerian distribution. This function $l_k(r)$ can be found by setting $\mathrm{d}V/\mathrm{d}r = 0$. After some algebra this leads to a quadratic equation whose correct physical root is

$$\psi = \{eB\,\Delta r - [(eB\,\Delta r)^2 - D(r^2\lambda'\,\Delta\lambda^{-1} + m)]^{1/2}\}D^{-1} \tag{2.6}$$

where we have used the notation

$$\Delta = (1 - 2mr^{-1}) \qquad \lambda' = 2B^2 r \qquad D = 2\lambda\lambda'\,\Delta + \lambda^2 mr^{-2} - \Delta\lambda^2 r^{-1}. \tag{2.7}$$

Further, we have divided equations (2.4) and (2.5) through by μ^2 and μ respectively, so that in (2.6) and all subsequent equations we are referring to the specific potential, momentum, charge, etc; $V_s = V/\mu^2$, $l_s = l_k/\mu$, $e_s = e/\mu$, $\psi_s = \psi/\mu$. We shall henceforth ignore the subscript 's'.

The marginally bound orbit can be found by setting $V = 1$. The binding energy then vanishes, as the binding energy b per unit mass is given by

$$b = 1 - V^{1/2}. \tag{2.8}$$

The marginally stable orbit is found by setting $\mathrm{d}l_k/\mathrm{d}r = 0$. Unfortunately, neither of these equations can be solved analytically, so we shall resort to expansions in two regimes of interest and shall then perform numerical calculations to provide values where neither expansion is valid.

We shall show below that under reasonable physical conditions in active galactic nuclei we would never expect $|Bm|$ to exceed 10^{-4}. Thus the magnetic field is weak (in geometrical units), as required. Under the circumstances of hydrodynamic accretion, where one has an essentially neutral plasma (only slightly affected by charge separation), one also has that $|eB| \ll 1$. (But see Lovelace (1976) or Blandford and Znajek (1977) for cases where this may not hold.) The other possibility, of less astrophysical interest but still of intrinsic importance, is that of test particles. Then we can have $e \gg 1$ and the possibility of $eBm \gg 1$.

First consider hydrodynamic flow with $Bm \ll 1$ and $eBm \ll 1$ (or $e\Phi \ll 1$ where r is $\mathrm{O}(m)$). Note that under these circumstances we expect $r_{\mathrm{mb}} \simeq 4m$, $r_{\mathrm{ms}} \simeq 6m$, and the last circular photon orbit, where l or V reach maxima, $r_{\mathrm{ph}} \simeq 3m$, while $l(r_{\mathrm{mb}}) \simeq 4m$, $l(r_{\mathrm{ms}}) \simeq \sqrt{12}m$, $b(r_{\mathrm{ms}}) \equiv b_{\mathrm{ms}} \simeq 1 - \sqrt{8/9}$, and we are interested in the amount and direction of the deviation from these values as functions of B and e. Expanding equation (2.6) and inserting it into (2.5) we find, to lowest appearing orders in Bm and eBm, that

$$l(r = 4m) = 4m(1 + 80B^2m^2 - 4eBm) \tag{2.9a}$$

$$V(r = 4m) = 1 + 128B^2m^2 - 8eBm \tag{2.9b}$$

$$b(r = 4m) = -64B^2m^2 + 4eBm \tag{2.9c}$$

and that

$$l(r = 6m) = \sqrt{12}m(1 + 204b^2m^2 - \sqrt{12}eBm) \tag{2.10a}$$

$$V(r = 6m) = 8/9 + (512/3)B^2m^2 - (16\sqrt{12}/9)eBm \tag{2.10b}$$

$$b(r = 6m) = 1 - \sqrt{8/9}(1 + 96B^2m^2 - \sqrt{12}eBm); \tag{2.10c}$$

furthermore

$$r_{\mathrm{ph}} \simeq 3m\left(1 + \frac{12B^2m^2}{1 - 51B^2m^2}\right). \tag{2.11}$$

Note that if $Bm > e/16$ then equation (2.9c) tells us that $r_{mb} > 4m$, and the binding energy is already negative at $r = 4m$. Equation (2.10c) shows that if $Bm > \sqrt{3}e/48$ then the binding energy at $r = 6m$ is less than it would be in the Schwarzschild case, and we therefore anticipate less efficient accretion. As both e and Bm are assumed to be small, we see that the changes brought by the magnetic field are small in either the positive or negative (if $Bm < e/16$ or $\sqrt{3}e/48$) direction. Although we do not find the actual values of r_{ms} and r_{mb}, we expect them to differ by $O[m \times \max(Bm, e)]$ from their Schwarzschild values.

Now let us consider the test particle case. Although $Bm \ll 1$ is still presumed valid, we can easily have $e > 1$ or even $e \gg 1$ (for an isolated proton $e/\mu = 1.112 \times 10^{18}$ in geometric units). In fact, we shall assume $eBm \gg 1$ and then perform an expansion. We now see that $r \simeq 3m$ causes no real divergence for l or V, but rather all of the interesting values lie close to $r = 2m$, the event horizon. Expanding equation (2.6) under these circumstances yields $\psi \simeq [2eB(r-2m)]^{-1}m$ to the lowest order, so that

$$l_k(r; e, B) \simeq [2eB(r-2m)]^{-1}m + eBr^2. \tag{2.12}$$

Differentiating (2.12) and setting the result equal to zero gives

$$r_{ms} \simeq 2m[1 + (4\sqrt{2}eBm)^{-1}] \tag{2.13a}$$

$$l(r_{ms}) \simeq (4eBm + \sqrt{2})m \tag{2.13b}$$

$$V(r_{ms}) \simeq 3(8\sqrt{2}eBm)^{-1} \tag{2.13c}$$

$$b(r_{ms}) \simeq 1 - \sqrt{3}(8\sqrt{2}eBm)^{-1/2}. \tag{2.13d}$$

We can also solve for the value of r_{mb} in this approximation:

$$r_{mb} \simeq 2m[1 + (8eBm)^{-2}]. \tag{2.14}$$

For test particles we see that r_{ms} is quite close to $2m$ if eBm is large, and consequently r_{mb} will be pushed even closer to the event horizon. It is intereting to note from equation (2.13d) that even though r_{ms} is close to r_{mb} the efficiency of accretion can approach unity. Finally we note a general property of equation (4) is that $V(r = 2m) = 0$, so in this case there is a steep spike in the potential curves for $2m < r < r_{mb}$.

3. Physical constraints on the field strength and charges

The conversion between the dimensionless variable Bm and physical units for the magnetic field is

$$B_G = \frac{c^4}{G^{3/2}M}Bm = 2.36 \times 10^{19}(M_\odot/M)(Bm)\,\mathrm{G}. \tag{3.1}$$

As DHV point out, their critical value for the field, above which no bound orbits exist, is $Bm \simeq 0.101$, and that corresponds to an outrageously high field of $\sim 10^{10}$ G even for a supermassive black hole of $10^8 M_\odot$. We now proceed to set a more reasonable constraint on Bm by demanding that the pressure P_B due to the magnetic field does not exceed the total (gas plus radiation) pressure in the accretion disc near the cusp. (An even more stringent limit, which could be reasonably argued for, is that $P_B < P_{gas}$ alone.)

To estimate values for the pressure in the disc we use the recent models of Wiita (1982) for the physical properties of thick PW accretion discs. It turns out that the largest pressures are generated for his $n = 3$ polytropic models and are given by

$$P_{\max}(r) = 5.25 \times 10^{32} Z_s^{-4} (1 + Z_s^{-1})^{-3} U^4(r) \text{ dyn cm}^{-2} \tag{3.2}$$

where $U(r) = 2[(r-1)^{-1} - (R-1)^{-1}]$, $R = (r^2 + z_0^2)^{1/2}$, z_0 is the half-thickness of the disc, given by equation (2) of Wiita (1982), and Z_s is the ratio of radiation pressure to gas pressure at the surface of the disc. All the lengths are in units of m. When we examine all the physically allowed models discussed in that paper we find that maximal values of U near the cusp are ~ 0.034 and that Z_s can only vary between $\sim 3 \times 10^2$ and $\sim 3 \times 10^3$ for large black holes, $M > 10^6 M_\odot$. Inserting the lowest value for Z_s and the highest for U into equation (3.2) tells us that for $r_{mb} < r_0 \leq r_{ms}$

$$P_{\max}(r_0) < 7 \times 10^{16} \text{ dyn cm}^{-2}. \tag{3.3}$$

Now demanding $P_B \approx B_G^2/8\pi < P_{\max}$ implies

$$B_G < 1.3 \times 10^8 \text{ G} \qquad \text{or} \qquad |Bm| < 5.6 \times 10^{-12} (M/M_\odot). \tag{3.4}$$

If $P_B < P_G$ is felt to be a preferable constraint, the result is reduced by another factor of $Z_s^{-1/2}$. Other polytropic indices produce limits on Bm more severe by at least a factor of five.

Thanks mainly to the constraint that the disc mass be less than the black hole mass, Wiita (1982) finds that the highest mass for which his models are valid is less than $10^7 M_\odot$. Inserting this value into equation (3.4) provides the self-consistent limit of

$$|Bm| < 10^{-4}. \tag{3.5}$$

Although one could conceivably create higher-luminosity discs around black holes with $M \sim 10^8 M_\odot$, the disc would have to be at least as massive as the central object; (3.5) is unlikely to be exceeded by more than an order of magnitude even in such non-self-consistent cases. In actuality, we expect that $|Bm|$ will be far less than 10^{-4} which relied upon extreme values of Z_s and U. For more typical values of these parameters we find $|Bm| < 10^{-7}$.

The value of the charge to be assigned is somewhat more arbitrary. We expect the plasma to be neutral on the whole, but complex plasma–dynamic effects should allow for some charge separation near the black hole so that the equivalent of a non-zero value of e is likely. But, under most circumstances we expect this effective charge/mass ratio to have a magnitude less than unity (in geometrical units). As the magnetic field structure has been so highly idealised, it does not seem worthwhile to attempt a detailed calculation for the effective charge, as our results cannot give more than a qualitative indication of what really goes on in the astrophysical setting. We resolve the uncertainty over what values of e to choose by picking a very wide range of values in calculating the numerical results.

Nevertheless, it is possible to argue more quantitatively that collective plasma effects, and therefore fluid behaviour, will dominate the motion in thick, supercritical accretion discs. Following Prasanna (1980) we assume that the plasma is adequately treated in the test particle approximation if the collision frequency of the particles is much less than the gyrofrequency, or, equivalently, that the mean free path of the particles is greater than the effective field variation length scale $|B/(\mathrm{d}B/\mathrm{d}r)|$. In our metric, $\mathrm{d}B/\mathrm{d}r \simeq 0$ so this condition is never likely to be satisfied and the fluid approach is clearly to be preferred. But if we become more conservative and compare the mean free path with the field variation obtained from a dipole field, we can use Prasanna's result (his equation (7.7)) that if

$$N < 5.5T^2, \tag{3.6}$$

where N is the particle number density and T the temperature in the plasma (in CGS units), then the test particle approximation may be valid. Except for $n = 0$ polytropes, the smallest values of the ratio N/T^2 occur on the surface of the disc. Thus we combine Wiita's (1982) equations (9) and (10) for the surface temperature and density to obtain

$$N/T^2 > 5 \times 10^8 (M/M_\odot)^{-4/15} Z_s^{-16/15} Q^{2/15}(r). \tag{3.7}$$

Because $\sim 0.03 < Q^{2/15}(r) < \sim 0.58$, we see that even for extreme, non-self-consistent models ($M = 10^8 M_\odot$, $Z_s = 10^4$) we have $N/T^2 > 10$, and, more typically (e.g. $M = 10^6 M_\odot$, $Z_s = 10^3$), $N/T^2 > 300$. Inside the disc this ratio soars to greater than 10^4–10^6, so that for any thick disc we must use the fluid approach and the effective e value should be small. However, for sufficiently sub-critical accretion it is possible for (3.6) to be satisfied, and then extremely high efficiencies could be allowed.

4. Discussion

In figure 1 we display l_k and V as functions of r for three sets of (B, e) values. For $Bm = 10^{-4}$ and $e = 4.8 \times 10^{-10}$ we have $1 \gg Bm \gg e/16$, so that we expect increases in $r_{\rm mb}$, $r_{\rm ms}$ and a decrease in b with respect to the Schwarzschild parameters; although these changes are pesent, they are too small to be discerned on the scale of the figure, so these broken curves are essentially the same as for the non-magnetic case. These very minor shifts are typical for the standard astrophysical situation. If $e < 0$, then the efficiency is invariably decreased.

For $Bm = 10^{-2}$ and $e = 1$ we have a situation where neither of our expansions is really applicable. But, since $Bm < e/16 < 1$, we expect that $r_{\rm mb}$ and $r_{\rm ms}$ would decrease while b would increase. As can be seen from the figure, this is so: $r_{\rm mb} = 3.78m$, $r_{\rm ms} = 5.73m$ and the binding energy at $r_{\rm ms}$ is some 7.9%, implying an efficiency about 38% higher than that for the Schwarzschild case. However, this field is too strong to be physically justified.

Finally we have plotted results for the case of a weak field, but high e (i.e. test particle): $Bm = 10^{-8}$, $e = 10^9$. Here $eBm > 1$ and we expect our second expansion to be nearly valid. We find $r_{\rm mb} = 2.05$, $r_{\rm ms} = 2.06$ and $b = 0.810$, a very substantial (13-fold!) increase in the conversion of mass to energy. Our results for large eBm are qualitatively very similar to those of Prasanna and Varma (1977) even though they use a dipole field. We refer the reader to their extensive tables and figures and their discussion of particle orbits for more details. In particular, in these cases we also find a second maximum in the effective potential that is greater than unity, implying that particles can be trapped between $r_{\rm mb}$ and an outer turning point which is located at $r > 20m$ in all cases. For our calculations of significance to actual accretion discs, these secondary humps in the potential do not always cross unity, and even when they do, the most negative value attained for the binding energy is so small that pressure and viscous forces should certainly be able to push the plasma over the hump, enabling it to approach from large radii and still be swallowed by the black hole.

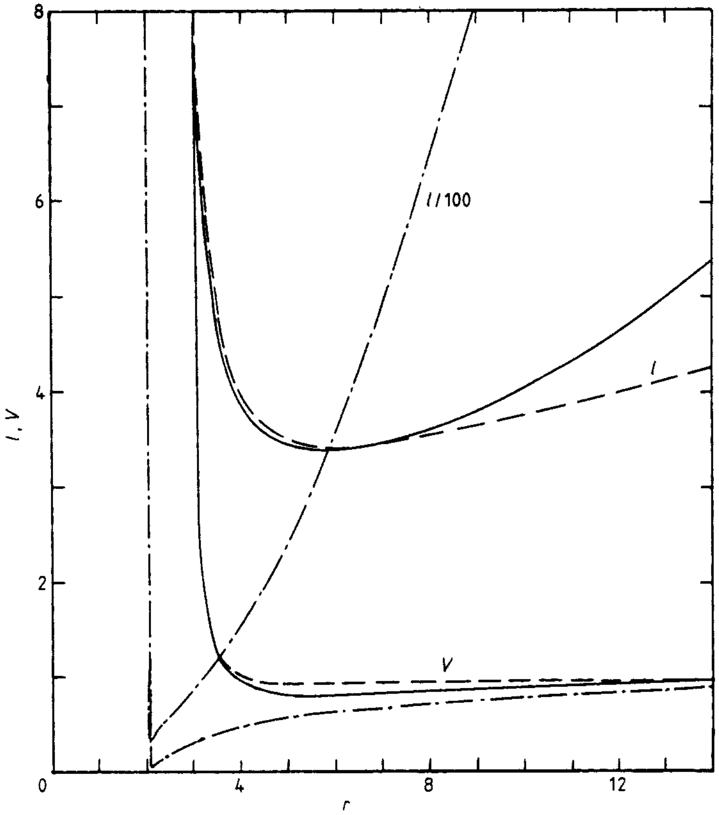

Figure 1. The effective potential V (in units of μ^2) and the Keplerian angular momentum distribution l (in units of μ). The three cases shown are: $Bm=10^{-2}$, $e=1$ (full curves); $Bm=10^{-4}$, $e=4.8\times10^{-10}$ (broken curves); $Bm=10^{-8}$, $e=1\times10^{9}$ (chain curves). The second case is indistinguishable from the pure Schwarzschild metric on the scale of this drawing. Note that for the last case $l/100$ instead of l is plotted and also that on this scale all the potential curves drop precipitously to zero at $r=2$ (not shown for clarity).

From equations (2.5) and (2.6) we see that l_k becomes imaginary as $r\to\infty$, but even before that, for $r>1/B$, $dl/dr<0$, implying that the disc cannot be stable. Physically this means that the assumption that the magnetic field is essentially uniform over a tremendous distance ($>10^4 m$) is untenable, and we must allow for a decrease in the field at least as rapid as $1/r$ at large radii for a self-consistent solution.

Table 1 lists the values of r_{mb}, l_{ms}, r_{ms}, b_{ms} and $\Delta b\equiv(b_{ms}-b_{sch})/b_{sch}$ for values of Bm from 10^{-10} to 10^{-2} and of e from $\sim10^{-34}$ to $\sim10^{18}$. It is clear that at sufficiently large eBm the efficiency of accretion can approach unity. However, when $Bm<10^{-4}$ and $e<1$, which should be relevant to accretion around supermassive black holes, the changes in efficiency never exceed 0.6%. We have also performed calculations that mock up a field that is not intrinsic to the geometry, i.e. we set $\lambda=1$ in the metric (2.1) but retain it in the electric potential (2.3). This implies that the field is being carried in by the accreted material, which is probably what happens in the physical situation. Of course we are still neglecting the real interaction of the field with the matter, which tends to compress the former away from the uniform structure assumed. However, since the physical constraint is applied near r_0, and the field should be near

Table 1. Parameters for marginally bound and marginally stable orbits.

Bm	e/μ	r_{mb}	r_{ms}	l_{ms}/μ	$b_{ms}/\mu c^2$	Δb
1(−10)†	1.00	4−O(−10)	6−O(−10)	3.464 102	0.057 190 96	O(−9)
	1.00(9)	3.058 279	3.982 675 21	3.522 333	0.215 474 65	2.767 634
	1.11(18)	2+O(−16)	2+5.2(−9)	4.448(8)	0.999 941 06	16.484 251
1(−8)	4.80(−10)	4+O(−16)	6+O(−16)	3.464 102	0.057 190 96	−O(−14)
	1.00(−2)	4−O(−10)	6−O(−10)	3.464 102	0.057 190 96	5.71(−9)
	1.00	3.999 9998	5.999 999 91	3.464 101	0.057 190 99	5.71(−7)
	1.00(2)	3.999 9994	5.999 999 43	3.464 090	0.057 194 22	5.71(−5)
	1.00(4)	3.996 8099	5.999 982 14	3.462 905	0.057 517 16	5.70(−3)
	1.00(9)	2.047 6584	2.055 654 34	4.343(1)	0.810 431 46	13.170 622
	1.11(18)	2+O(−20)	2+5.2(−11)	4.448(10)	0.999 993 93	16.485 178
1(−6)	4.80(−10)	4+O(−12)	6+O(−12)	3.464 102	0.057 190 96	−O(−10)
	1.00(−2)	3.999 9998	6.000 000 44	3.464 102	0.057 190 99	5.69(−7)
	1.00	3.999 9684	5.999 999 90	3.464 090	0.057 194 22	5.71(−5)
	1.00(2)	3.996 8099	5.999 981 85	3.462 905	0.057 517 61	5.70(−3)
	1.00(4)	3.753 4894	5.841 397 47	3.375 860	0.086 163 09	0.506 586
	1.00(9)	2+O(−6)	2.000 577 18	4.004(3)	0.980 388 20	16.142 363
	1.11(18)	2+O(−24)	2+5.2(−13)	4.448(12)	0.999 994 11	16.485 271
1(−4)	4.80(−10)	4.000 0055	6+O(−8)	3.464 109	0.057 190 05	−1.58(−5)
	1.00(−2)	3.999 9735	5.999 966 27	3.464 097	0.057 193 32	4.12(−5)
	1.00(0)	3.996 8155	5.999 948 25	3.462 912	0.057 516 23	5.69(−3)
	1.00(2)	3.753 4926	5.841 373 63	3.375 866	0.086 162 33	0.506 573
	1.00(4)	2.345 0897	2.439 917 75	7.277 229	0.516 891 46	8.037 993
	1.00(9)	2+O(−10)	2.000 005 59	4.000(5)	0.998 034 44	16.450 913
	1.11(18)	2+O(−28)	2+5.2(−15)	4.448(14)	0.999 999 94	16.485 280
1(−2)	1.38(−34)	4.054 6859	5.733 992 39	3.530 255	0.048 373 18	−0.154 376
	4.80(−10)	4.054 6859	5.733 992 39	3.530 255	0.048 373 18	−0.154 376
	4.80(−10)‡	4.013 0634	6.044 326 51	3.451 585	0.053 740 20	−0.060 342
	1.00(−2)	4.051 0844	5.734 057 18	3.528 992	0.048 710 97	−0.148 274
	1.00	3.783.9188	5.638 201 88	3.432 165	0.078 642 46	0.375 086
	1.00‡	3.760 6017	5.870 287 88	3.365 692	0.082 922 88	0.450 152
	1.00(2)	2.345 4126	2.440 266 30	7.275 317	0.516 402 83	8.029 449
	1.00(4)	2.004 9772	2.005 753 72	4.033(2)	0.938 144 58	15.403 722
	1.00(9)	2+O(−14)	2.000 000 06	3.998(7)	0.999 803 315	16.481 842
	1.11(18)	2+O(−32)	2+5.1(−17)	4.446(16)	0.999 999 99	16.485 281

† Numbers in parentheses are powers of ten.
‡ Calculated for an external magnetic field ($\lambda = 1$ in the metric).

its maximum there, our assumption is not so bad, at least as far as the efficiency is concerned. The differences between the values of the quantities given in the table calculated in this fashion and those from our primary approach never differed by more than 1 part in 10^5 except for $B = 10^{-2}$ and $e < 1$. Two such extrinsic field cases are also listed in table 1.

The above manoeuvre does not show how the field will actually be carried towards and, presumably, concentrated around the black hole by the infalling material (Bisnovatyi-Kogan 1979), and our simple form for the field is a major weakness of our approach. Very complex, but self-consistent calculations, along the lines of Prasanna and Chakraborty (1981), but extended to thick discs, are what are really required, and Prasanna (private communication) is attempting them. Other limitations are our restricting our analysis to the equatorial plane and our consideration of only the 'Keplerian' angular momentum distribution, as thick discs must have non-Keplerian distributions. However, the efficiency should still be determined by the Keplerian value of $l(r_0)$ on the equator (PW). We have also neglected radiation losses which could be important in the test particle situation, but only strengthen our result for the plasma case. Despite these approximations, we are confident that our primary result—that the changes in accretion efficiency due to a physically allowed magnetic field in an accretion disc are small—is essentially valid. Thus, if one is searching for a way to increase the luminosity of a thick disc significantly, or to improve the collimation of the beams emerging from one, this particular effect of magnetic fields upon accretion is not likely to be the way to go about it.

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